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Long relaxation times and tilt sensitivity in Rayleigh Bénard turbulence

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Abstract. We study the influence of a small tilt angle ($0 < \theta < 3 \times 10^{-2}$rd) on the Nusselt number in a 1/2 aspect ratio Rayleigh-Bénard cell, at high Rayleigh number ($5 \times 10^{11} < Ra < 4 \times 10^{12}$). The small decrease observed is interpreted as revealing a two rolls structure of the flow. Transitions between different global flows are also observed, on very long times, comparable to the diffusion time on the whole cell. The consequence is that the Nusselt number observed in most high Ra experiments should significantly depend on initial conditions.

PACS. 92.60.Ek Convection, turbulence, and diffusion – 47.27.Te Convection and heat transfer – 44.25.+f Natural convection

1 Introduction

The paradigm of Rayleigh Bénard convection assumes infinite horizontal plates between which the fluid is driven by the plates' temperature difference. The control parameters are the Rayleigh number and the Prandtl number,

$$Ra = \frac{g\alpha \Delta T h^3}{\nu\kappa} \text{ and } Pr = \frac{\nu}{\kappa} \qquad (1)$$

where ν is the kinematic viscosity, κ the heat diffusivity, g the gravity acceleration, α, the isobaric thermal expansion coefficient, ΔT, the temperature difference between the plates, and h, the height of the cell.

Obviously, experiments and numerical simulations use a finite dimension d for the plates. The role of the aspect ratio $\Gamma = d/h$ is well documented [1–3]. But other parameters can have some influence. The influence of a finite conductivity for the walls has been recently examined in details [4–6]. The plates properties, namely their capacity to maintain a uniform and constant temperature, has also been discussed [7–9]. An eventual tilt of the plates from the horizontal is also a parameter to consider.

Very few papers studied the influence of a tilt [10]. For instance, reference [10] only look at a 10° tilt. Some comments appear in various works on the weakness of the effect [11], but without real study. Moreover, some experiments [4] and numerical studies [12] showed that different large flow structures are possible with the same values of parameters Ra and Pr. A tilt can be seen as an external field acting on the large scale flow, as used for instance in [10]. Thus, a strong motivation for the present work is to have a precise study of the influence of this parameter on the effective conductivity of the cell, the Nusselt number

$$Nu = \frac{Qh}{\lambda S \Delta T} \qquad (2)$$

where Q is the heat power supplied to the bottom plate, λ, the thermal conductivity of the fluid, and S the plates area. Specifically, we attempt to quantify here the influence of a tilt at high Rayleigh numbers ($Ra \simeq 10^{12}$).

An important and unexpected byproduct of this study concerns the global dynamics of the flow, particularly the times involved for a transition from a metastable convection state to another.

The paper organizes as follows. We first present the experimental set-up and the characteristics of the cell. Then we discuss the influence of the tilt. Finally we report on the phenomenon of transition between states, we discuss the times involved, and their implication for other works.

2 Experimental set-up

Our cell [7] is cylindrical, $d = 50$ cm in diameter, $h = 1$ m high (see Fig. 1). The cylindrical wall is made of stainless steel, 2.5 mm thick. Plates are made of copper, 3 cm thick, nickel plated. The working fluid is deionized water, which was made free of gas in situ by boiling several hours under partial vacuum. The bottom plate has a heating wire, 13.55 Ω, embedded as a spiral. It lays on an isolating PTFE plate, 2 cm thick which is supported by a square

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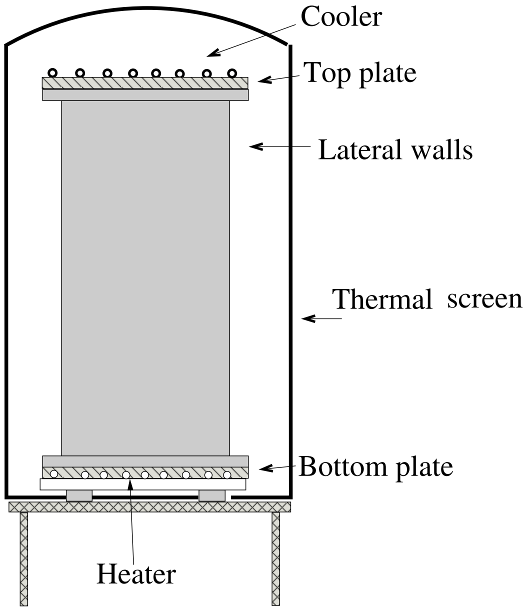

Fig. 1. Schematic view of the set-up.

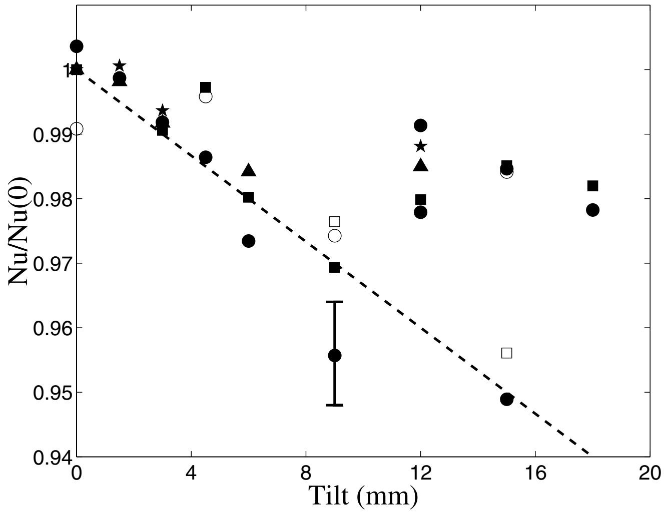

Fig. 2. Ratio between the Nusselt number for the tilted cell and for the horizontal cell one for the same Ra. ∘: $Ra = 4 \times 10^{12}$, □: $Ra = 2 \times 10^{12}$, △: $Ra = 10^{12}$, ⋆: $Ra = 5.2 \times 10^{11}$. Full symbols: first stable value. Open symbols: final stable value. Dashed line: expected dependence for the two rolls flow.

aluminium table, 3 cm thick, through eight hollow stainless steel feet. These feet go through a copper thermal screen which surrounds the whole cell. Heat leaks from the bottom plate are modelized by linear thermal links to the screen, the aluminium table, and the top plate (respectively 0.7, 0.5 and 0.1 W/°C). Calibration of these links allows to estimate the heat leaks within 0.5 W. Further correction of wall effect has been applied, following the works [4–6], using formulas proposed by [4].

The top plate is cooled through two counter flow spiral tubes, 1 cm inner diameter, weld on the plate. Temperatures are measured through type K thermocouples. Additional control platinum thermometers are glued on both plates. The water cooling the top plate is itself cooled through heat exchange with fresh water, and temperature regulated via a 1 kW heater-cooler bath. The control sensor is on the top plate. To minimize uncontrolled heat leaks, the screen is regulated at the mean temperature between the two plates. The aluminium table is also temperature regulated.

All the related study has been made with water at an average temperature of 77 °C, which also was the temperature of the screen. The Prandtl number at this temperature is close to 2. The interior of the cell is totally free of any thermometer or other apparatus.

3 Influence of a tilt

The study has been made as follows. We begin by checking the horizontality of the bottom plate carefully, and measure the Nusselt number Nu for five values of the Rayleigh number: $Ra = 4\times10^{12}$, 2×10^{12}, 10^{12}, 5.2×10^{11}, 2.6×10^{11}, corresponding to differences in temperature between the plates ΔT going from 31 °C to 2 °C. For the highest Ra, the total injected power is close to $Q = 4$ kW, corresponding to $Q/S \simeq 20$ kW/m^2.

Then, we lower the feet of the square aluminium table on one side, by steps. A thermocouple is glued on the stainless steel wall on the same side, at midheight. We initially planned to measure the Nusselt number at each step for all the above mentionned Rayleigh numbers. However, we rapidly realized that, due to the small variations we observed, the smallest Ra could not allow a sufficiently high accuracy for any conclusion to be drawn, and we occasionally skipped them. At the end of each series of Rayleigh numbers, we let the cell for two days, without heating nor cooling, only the screen being maintained at 77 °C. Then we move to the new value of the tilt, just before to begin the series with the highest Ra.

The results are presented on Figure 2. We measure the slope by the height we lower the feet, in millimeters: successively 1.5, 3, 6, 12, 18, 15, 9, 4.5, and 0 mm. As the distance between the feet of the aluminium table is 600 mm, dividing these values by 600 gives the angles in radians. After 5 months of measurements, the Nu values we find for the horizontal plates are the same than the original ones within less than 1%. The error bar we deduce from the dispersion of values of the "middle" thermocouple (see above) is ±0.8%. Indeed, these values show no clear tendency with the tilt angle. We thus use as error bar two standard deviations of these values, i.e. 0.25 °C.

In Figure 2, the Nu values are normalized by the initial, horizontal plates value for each Ra, which are, respectively, $Nu = 830$, 658, 523, 413, and 348. Each symbol shape corresponds to a value of Ra. The difference between open and full symbols refers to transitions we discuss in the following section. We shall now stress on the obvious information we can get from the results. First the influence of the tilt is always to lower the Nusselt. Second this influence is small. Indeed, it is smaller than the absolute precision in most of the previous works. Thus an eventual difference between such works cannot be attributed to a tilt of one cell. Specifically, the influence of an eventual tilt cannot be invoked to explain the differences between high Rayleigh number experiments [2,10,13–16].

While it is coherent with various remarks in previous papers [10,11], such a small influence of a tilt was not a priori obvious. One could have thought of a visible change as soon as the height one side is lowered is comparable to the thermal boundary layer thickness. For our highest Ra, the boundary layer thickness is $h/2Nu = 0.6$ mm. In the last section, we discuss of a possible interpretation of these small values.

The third remark we can do is that two groups seem to appear within these data. The dashed line, which will be explained later, helps in stressing on the first group, which shows a small, but clear, approximately linear dependence with the tilt angle. The second group, which appear only for a tilt larger than 9 mm, shows no clear tendency. With only these points, we could have concluded that the Nusselt number does not depend on the tilt angle. In the theoretical interpretation, we shall suggest that these two groups correspond to two different structures for the global flow.

4 Transition times

As rapidly mentionned above, we occasionally observed that transitions can occur, without any external intervention, so that ΔT and thus Nu to change, at constant injected power. Figure 3 presents such a transition, which occurred for the highest Rayleigh number, and horizontal plates, in the final run. For a clear observation of this transition, we maintained the power constantly applied for more than one month. During the first two weeks, ΔT was nearly constant. Then, a progressive change occurred, whose completion asked for nearly 4 days. ΔT then remained constant for 3 weeks before we stopped.

Note that, at the beginning of one run, a stable ΔT is obtained within a much shorter time constant τ_o, of order 1 h (see Fig. 4). This is coherent with $\tau_o = h^2/(2\kappa Nu)$ corresponding to the enhanced convective conductivity. Even much shorter equilibrium times, of the order of the boundary layer diffusion time (τ_o/Nu) can be obtained if the middle cell temperature is maintained constant. Here, the transition time is 100 h, only 8 times smaller than the non convective diffusion time $h^2/(4\kappa)$. Once scaling is made, one realizes that this is larger than most of the measurement times in other systems.

For smaller Ra, and helium gas in the same aspect ratio, Roche et al. [17] observed bimodal behaviour of the Nusselt number. Verzicco [12] showed that, for this aspect ratio, two main flows are possible, one with a single roll covering the whole height of the cell, the other with two rolls. Here, for the first time, we observe the dynamics of the transition between two flows, which differs from the random precession of the rolls observed by Cioni et al. [18], and Niemela et al. [19]. It shows how a large characteristic times spectrum can be generated in such systems.

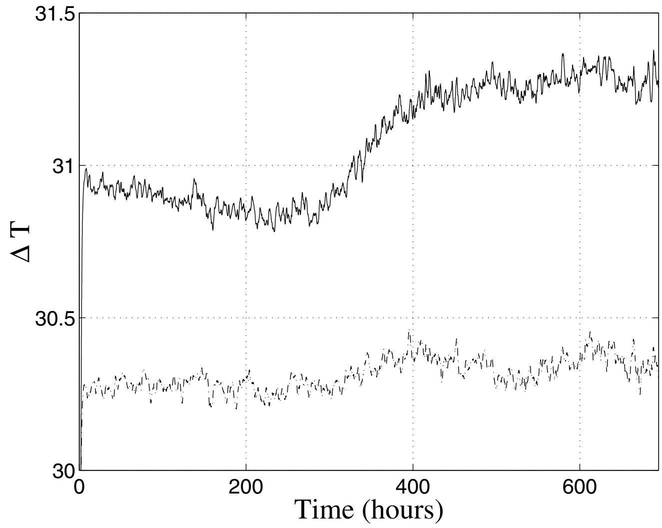

Fig. 3. Solid line: the plate temperature difference versus time, showing the transition. Dashed line: the middle temperature difference with the cold plate is poorly influenced by the transition, and is a test of the temperature measurement stability. It is shifted by 14.5 degrees for convenience.

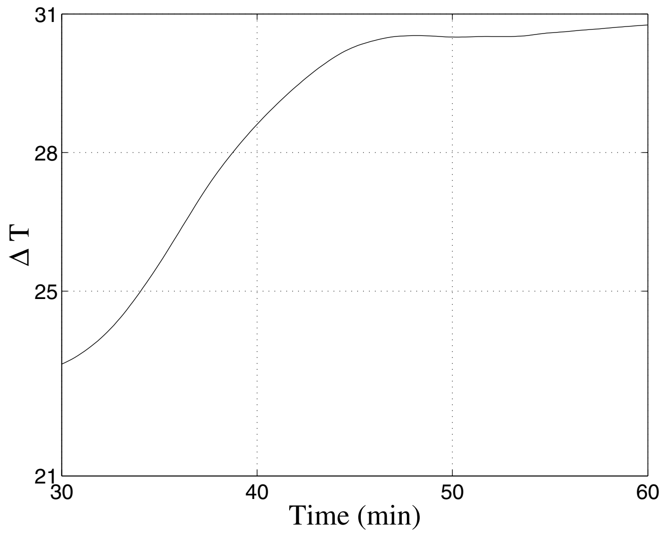

Fig. 4. The initial time constant is much shorter than the transition time.

Similar transitions have been observed with the inclined cell too. In Figure 2, we signal the final state with an open symbol. Other transitions could have been missed due to a too small measurement time, even if all the measurements took at least two days. Each observed transition took a similar characteristic time: 4 days. Note that, precisely due to these long characteristic times, we cannot conclude that the secondly observed state is a stable one.

5 Theoretical interpretation

In this section, we propose a partial interpretation of our results. Shortly, our interpretation starts from the observation of Verzicco [12], that two flows are possible, with one roll or two rolls in the height of the cell. We argue that the two rolls state is sensitive to the tilt, and that the one roll state, nearly insensitive to the tilt, is favorized at large tilt.

The first remark is that two rolls in the height must be counter-rotative. Thus, with an inclined cell, if the flow goes down when running along the top (cold) plate, it must go down too when along the bottom (hot) plate, which is against gravity. On the contrary, a single roll can be in agreement with gravity on both plates.

Note that a single roll, favorized by gravity, should not give a higher Nusselt, when the cell is tilted. For instance, it is known [20] that a horizontal temperature gradient (vertical plates) does not give a higher Nu than a vertical gradient, for the same high Ra. The reason is a tendency of the flow to localise itself close to the walls, with a stratified center. We think that the points for the tilt larger than 9 mm, in the upper part of the Figure 2, for which the decrease in Nu is small, correspond to such a single roll flow.

In the same spirit, we shall assume that, for the two rolls state, the favorized roll keeps the same effective thermal conductivity. To estimate to what extent the defavorized roll has its conductivity lowered, we shall first calculate the velocity u' imposed to the fluid by the gravity component parallel to the plate ($g\theta$ where θ is the tilt angle). The force per unit area is $\rho\delta g\theta\alpha\Delta T/2$, with ρ the fluid density, and $\delta = h/2Nu$ the thickness of the thermal boundary layer. The viscous stress is $\rho\nu u'/\delta$. Thus:

$$u' = \frac{\delta^2 g\theta\alpha\Delta T}{2\nu} \text{ and } Re' = \frac{u'h}{\nu} = \frac{Ra\theta}{8PrNu^2} \qquad (3)$$

where $Pr = \nu/\kappa$ is the Prandtl number. u' corresponds to a drift velocity of the fluid close to the plate. Its exact effect on the Nusselt number is not easy to determine, but should be in order of magnitude measured by the ratio between u' and the roll velocity u:

$$\frac{Nu(0) - Nu(\theta)}{Nu(0)} = \frac{Re'}{Re}. \qquad (4)$$

The dependence of $Re = uh/\nu$ versus Ra and Pr has been estimated in several recent works [2,21,22]. They are not in full agreement, but all give the same order of magnitude in our range. For instance, Chavanne et al. [2] propose: $Re \simeq 0.206Ra^{0.49}Pr^{-0.7}$, which gives:

$$\frac{Re'}{Re} \simeq 0.6\theta\frac{Ra^{0.51}}{Pr^{0.3}Nu^2} \simeq 2\theta. \qquad (5)$$

The corresponding dependence appears as a dashed line on Figure 2, showing that the order of magnitude is good. In the same spirit, we replaced the expression $Ra^{0.51}/Pr^{0.3}Nu^2$ found above by a constant as its dependence with Ra is small.

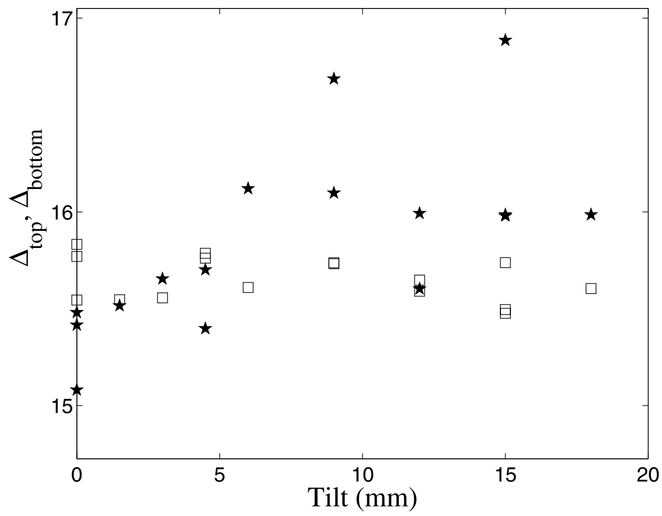

Fig. 5. □: Δ_{top}, the temperature difference between the middle of the cell and the top plate. ⋆: Δ_{bottom}, the temperature difference between the bottom plate and the middle.

Thus the lower points in our diagram, Figure 2, would correspond to a two rolls structure, with a roughly linear dependence with the tilt angle, due to one roll being defavorized by the gravity. The upper group would correspond to a single roll structure. Confirming this interpretation, the thermocouple glued on the middle of the wall shows that its temperature difference with the top plate does not depend on the tilt angle. Figure 5 compares this difference, which we call Δ_{top}, with its complement, Δ_{bottom}, the difference between the bottom plate temperature and the middle one. Δ_{bottom} has much larger variations with the tilt angle, suggesting that this tilt influences only the lower part of the cell.

If our interpretation is correct, the most probable configuration at zero tilt for our Ra range is the two rolls one, in agreement with the numerical studies [12].

6 Conclusion

In this paper, we quantify the influence of a tilt of the cell on the Nusselt number. This dependence is small, but a tilt always lowers Nu. A finite tilt can also make a selection between different possible flows. A transition between two different flows, which we observed here for the first time, takes very long to complete, of the order of the diffusion time on the whole cell. An ergodic statistics of the possible flows is probably out of reach experimentally for $Ra > 10^{11}$.

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References

1. E. Siggia, Ann. Rev. Fluid Mech. **26**, 137 (1994)
2. X. Chavanne, F. Chillà, B. Chabaud, B. Castaing, B. Hébral, Phys. Fluids **13**, 1300 (2001)
3. G. Ahlers, X. Xu, Phys. Rev. Lett. **86**, 3320 (2001)
4. P.-E. Roche, B. Castaing, B. Chabaud, B. Hébral, J. Sommeria, Eur. Phys. J. B **24**, 405 (2001)
5. G. Ahlers, Phys. Rev. E **63**, 15303 (2001)
6. R. Verzicco, J. Fluid Mech. **473**, 201 (2002)
7. S. Chaumat, B. Castaing, F. Chillà, *Advances in Turbulence IX, Proc. of the Ninth European Turbulence Conference*, edited by I.P. Castro, P.E. Hancock, T.G. Thomas (CIMNE, Barcelona, 2002), p. 159; F. Chillà, M. Rastello, S. Chaumat, B. Castaing, Phys. Fluids **16**, 2452 (2004)
8. J.C.R. Hunt, A.J. Vrieling, F.T.M. Nieuwstadt, H.J.S. Fernando, J. Fluid Mech. **491**, 183 (2003)
9. R. Verzicco, *High Rayleigh Number Convection Workshop, Lorentz Center, Leiden, June 2003*; R. Verzicco, Phys. Fluids **16**, 1965 (2004)
10. S. Ciliberto, S. Cioni, C. Laroche, Phys. Rev. E **54**, R5901 (1996)
11. P.-E. Roche, B. Castaing, B. Chabaud, B. Hébral, Phys. Rev. E **63**, R045303 (2001)
12. R. Verzicco, R. Camussi, J. Fluid Mech. **477**, 19 (2003); R. Verzicco, R. Camussi, J. Fluid Mech. **383**, 55 (1999)
13. X. Chavanne, F. Chillà, B. Castaing, B. Hébral, B. Chabaud, J. Chaussy, Phys. Rev. Lett. **79**, 3648 (1997)
14. X.Z. Wu, A. Libchaber, Phys. Rev. A **45**, 842 (1992)
15. J.J. Niemela, L. Skrbek, K.R. Sreenivasan, R.J. Donnelly, Nature **404**, 837 (2000)
16. J.J. Niemela, K.R. Sreenivasan, J. Fluid Mech. **481**, 355 (2003)
17. P.-E. Roche, B. Castaing, B. Chabaud, B. Hébral, Europhys. Lett. **58**, 693 (2002)
18. S. Cioni, S. Ciliberto, J. Sommeria, J. Fluid Mech. **335**, 111 (1997)
19. K.R. Sreenivasan, A. Bershadskii, J.J. Niemela, Phys. Rev. E **65**, 056306 (2002)
20. A. Belmonte, A. Tilgner, A. Libchaber, Phys. Rev. E **51**, 5681 (1995)
21. S. Lam, X.-D. Shang, S.-Q. Zhou, K.-Q. Xia, Phys. Rev. E **65**, 066306 (2002)
22. S. Ashkenazi, V. Steinberg, Phys. Rev. Lett. **83**, 3641 (1999)